Juggling Bits

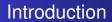
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Marco Gallotta Juggling Bits



- Most useful optimisations are high-level
- Bit manipulation is one of the most effective low-level optimisations
- Potential order-of-magnitude improvement in speed and size
- Can simplify code

















• Truth table for ! (not), && (and), |+| (or) and ^ (xor):

A	В	!A	A && B	A B	A ^B
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	1	1
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- Bit-wise operations do the same, but operate on each bit separately
- If A is 1100 and B is 1001, then:
 - ~A = 11110011 (assuming A is one byte)
 - A & B = 1000
 - A | B = 1101
 - A ^B = 0101



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- Two more operators: A « B and A » B
- Shift all bits in A and shifts them B positions to the left («) / right (»)
- Non-negatives are padded with zeros
- Equivalent to multiplyication («) / integer division (») by 2^B
- Most common use is 1 « X, which is a number with only bit X set



- We can use an integer to represent a subset of a set of up to 32 values
- A 1 bit represents a member in the subset, a 0 bit a member that is absent
- We have the following simple operations on subsets:

Union		Intersection	A & B
Subtraction		Negation	ALL_BITS ^A
Set bit	A = 1 « bit	Clear bit	A &= ~(1 « bit)
Test bit	(A & 1 « bit)	!= 0	



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- Easy to iterate over all subsets
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- Value of lowest bit: x & ~ (x 1)
- Index of highest/lowest bit: looping requires only two iterations on average
- GCC built-in functions:
 - __builtin_ctz (count trailing zeros)
 - __builtin_clz (count leading zeros)
 - Undefined for zero
- Check if number is a power of 2: clear lowest bit and check if result is 0
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- Very easy to make mistakes with bits
- A « B and A » B use only the last 5 bits of B
 - Shifting by 32 bits does nothing!
- & and | operators have lower precedence than comparison operators





- STL offers a conveniant data structure, bitset<N> in header bitset
- Optimised for space: each element occupies only one bit
- Advantages:
 - Easier than array of integers when more than 64 bits required
 - Some handy methods
- Disadvantages:
 - Need to know size in advance: template parameter N
 - No iterators: ++ and --
- Operators: &, |, ^, «, » and their &= equivalents
- Operators: ~, ==, != and [] to access a bit
- Methods: set, flip, any, none, to_ulong
- Documentation:

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http://www.sqi.com/tech/stl/bitset.html Marco Gallotta Juggling Bits



Example

```
#include <bitset>
#include <iostream>
using namespace std;
int main() {
  bitset <8> a(100 ul), b(string("1001110"));
  cout << a << "_" << b << endl;
  cout << (a&b) << "..." << (a|b) << "..." << (a^b)
      << endl;
  a[0] = 1; cout << a << endl;
}
produces:
01100100 01001110
01000100 01101110 00101010
                                                 ъ
```



Sample Problems

- Bit-sets and bit count: http://www.topcoder.com/ stat?c=problem_statement&pm=6725&rd=10100
- All subsets: http://www.topcoder.com/stat?c= problem_statement&pm=6095&rd=9917
- Bit-sets with trick iteration:

http://www.topcoder.com/stat?c=problem_
statement&pm=6475&rd=9988

• No adjacent bits: http://www.topcoder.com/stat? c=problem_statement&pm=6400&rd=10000







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